Machinery Messages

Case History

Component identification of gear-generated spectra

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This article discusses the five fundamental frequencies generated by a pair of gears in mesh. It also discusses two little-known frequencies in-depth: assembly phase passage frequency and tooth repeat frequency.

Gear drive systems range from simple two shaft pinion and gear combinations to multiple shaft locked train gear systems such as are found in a shipboard main propulsion reduction gear. The increased power of prime movers and the higher rotational speeds involved have subsequently increased the speed and power requirements of gear drives common to industry. Gear drives are commonly found in compressor, fan and pump drive systems using either electric motor or internal combustion engine prime movers. Many drives exist that provide reliable operation at pitch line velocities of 35,000 ft/min (177.8 m/sec), rotational speeds of 40,000 rpm and power transmission capabilities in excess of 30,000 HP (22.4 Mw). This level of complexity requires that certain operational parameters such as temperature, pressure and vibration be continuously monitored to provide long-term reliable operation.

It is far beyond the scope and intent of this article to discuss the design aspects of the gear drives common to industry in detail and the specific manner that certain design aspects may impact the vibratory signature of a given gearset. The purpose of this article is to present a systematic approach to calculating the five discrete frequencies that are generated by a meshing gearset as well as a basic understanding of how these frequencies are generated.

The basic gearset

This article will examine a gearset involving two toothed members. The formulae for the fundamental spectra components are thus applicable to spur, helical and bevel gearing. The gearset will be composed of a pinion and gear in mesh. In accordance with generalized industry terminology, the smaller of the two members is termed the pinion and the larger is termed the gear. The following five items need to be determined on a pair of meshing gears to establish the five discrete frequencies generated by the gearset and thus, by summation, the signature of the gearset.

- Number of teeth on the pinion. .... (N_p)
- Pinion speed, rpm. .... (R_p)
- Number of teeth on gear. .... (N_g)
- Gear speed, rpm. .... (R_g)
- Ratio, N_g/N_p or R_p/R_g. .... (M_g)

Suitable application of these five items will lead to the following five frequencies. Each of these items will be discussed separately.

- Gear rotational frequency, Hz. .... (f_{rg})
- Pinion rotational frequency, Hz. .... (f_{rp})
- Mesh frequency, Hz. .... (f_m)
- Tooth repeat frequency, Hz. .... (f_{tr})
- Assembly phase passage frequency, Hz. ..... (f_a)

Gear rotational frequency

Gear rotational frequency is simply the rotational velocity of the gear expressed as revolutions per second of time or Hertz. Remember that this will be the frequency at which a specific tooth on the gear will enter mesh. The distinction is important. Without suitable instrumentation, an unbalanced gear and a gear with a single damaged tooth may generate very similar spectra. Mathematically the gear rotational frequency is: ►

$$fg = \frac{Rg}{60} \text{ (Hz)}$$

Pinion rotational frequency

The comments relative to gear rotational frequency are applicable to the pinion and need not be repeated. Mathematically, pinion rotational frequency is:

$$f_{rp} = \frac{R_p}{60} \text{ (Hz)}$$

Mesh frequency

Mesh frequency is the rate at which tooth pairs contact as they pass through mesh, expressed in Hz. In most petrochemical applications of gearing, mesh frequency will be in the kilohertz region. Mathematically, mesh frequency can be expressed as a function of the number of teeth on the pinion or gear and the respective rotor speed:

$$f_m = f_{rp} \times N_p = f_{rg} \times N_g \text{(Hz)}$$

For example, a gearset using a gear with 100 teeth and rotating at 1800 rpm will generate a mesh frequency of 3000 Hz.

$$f_m = 30 \times 100 = 3000 \text{(Hz)}$$

Assembly phase passage frequency

A given pair of mating gears may exhibit several unique phases of assembly. The concept of "phase of assembly" is perhaps best explained through the use of an example. Consider a gearset with a gear having 15 teeth and a pinion of 9 teeth. From a gear design standpoint, this particular tooth combination would offer some unique vibration characteristics. For purposes of example, the combination of 15 and 9 is convenient.

Assume that our 15 by 9 tooth combination has every tooth numbered and at the time of the original assembly gear tooth #1 is in mesh with pinion tooth #1. With this assembly phase, gear tooth #1 will mesh ***only*** with pinion teeth #1, #7 and #4. Gear tooth #1 will not mesh with the remaining six teeth of the pinion unless the drive is built with the teeth indexed to some combination other than gear tooth #1 to pinion tooth #1.

Building the gearset with gear tooth #1 in contact with pinion tooth #2 defines a second unique assembly phase in which only pinion teeth #2, #8 and #5 will contact gear tooth #1. The third and final assembly phase is gear tooth #1 meshing with pinion teeth #3, #9 and #6. Thus, three assembly phases exist for a gearset with a 15 by 9 tooth combination. These phases are shown pictorially in Figure 1.

Mathematically, the number of unique assembly phases (N_a) in a given tooth combination is equal to the product of the prime factors ***common*** to the number of teeth in the gear and pinion. The numbers 15 and 9 have the common prime factor of 3. Therefore, three assembly phases exist.

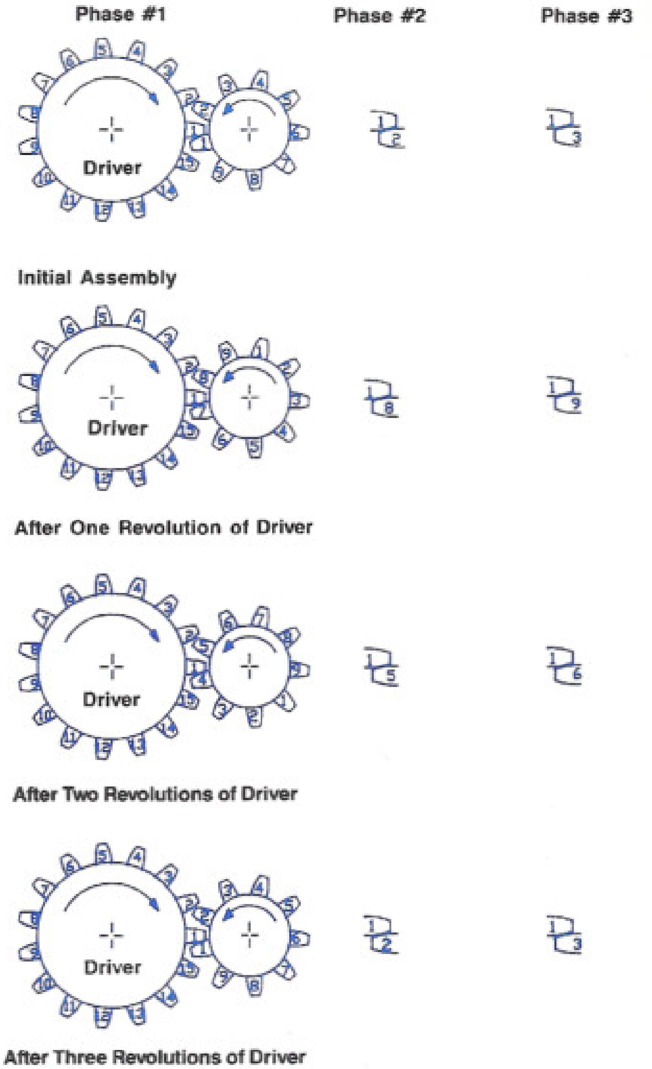

Figure 1
Assembly phases

The number of assembly phases determines the distribution of wear between the teeth of the gear and pinion. In our gearset example, it was shown that any single tooth ***on the gear only meshes with three pinion teeth.*** In a similar manner, it can be shown that a single tooth ***of the pinion*** will only mesh with five teeth of the gear. As the gear and pinion wear from service, each must accommodate the faults, however small, of the mating rotor. In the 15 by 9 tooth combination, the pinion will develop three distinct patterns of wear. See the following table:

Gear Tooth Numbers	Will Contact	Pinion Tooth Numbers
1-10-4-13-7		1-7-4
2-11-5-14-8		2-8-5
3-12-6-15-9		3-9-6

Note that pinion teeth #1, #7 and #4 only contact gear teeth #1, #10, #4, #13 and #7 and thus will develop a wear pattern dependent only on the gear teeth noted. Further examination will yield the remaining two wear patterns that could be generated as a result of a particular assembly phase. The effect of the wear patterns is to generate a 3X component on each rotor. In general (N_g/N_a) gear teeth will contact (N_p/N_a) pinion teeth and generate (N_a) wear patterns. The relative amplitude of the higher order component to the 1X component is a function of the degree of wear and extent that wear has changed tooth spacing and tooth profile accuracy.

Another effect of "phase of assembly" may exist in a gearset that has undergone disassembly for internal inspection or bearing replacement. In the 15 by 9 teeth example, the gearset was assembled with gear tooth #1 in contact with pinion tooth #1. If this gearset is now assembled with gear tooth #1 in contact with pinion tooth #2, new wear patterns will need to be established. During this phase of operation, the normal mesh frequency will be evident as well as a component at 1/3 of mesh. This component is generated as a result of the complex meshing action that occurs as a result of the previous wear patterns interacting with the "new" wear patterns generated. This may not be the only frequency component in evidence. The meshing process under such conditions involves both amplitude and frequency modulation of mesh frequency with many sum and difference frequencies apparent in the spectrum.

In summary, the assembly phase passage frequency is expressed as:

$$f_a = \frac{f_m}{N_a} \text{ (Hz)}$$

If you understand the concept of assembly phase passage frequency, it will be apparent why it is desirable to select tooth combinations that ***do not*** share common prime numbers between the gear and pinion. In such a case, a given gear tooth will contact every pinion tooth during operation. This is called a "hunting tooth" combination and generally will be found in most precision gear drives.

Due to the assembly phase passage frequency, it is an excellent practice to mark gearsets being disassembled to ensure the same phase of assembly when rebuilt. Normally this can be accomplished by marking two adjacent gear teeth with an "X" and the pinion tooth that falls between them with an "O."

Tooth repeat frequency

Referring to our 15 by 9 tooth gearset example and Figure 1, you will note that every three revolutions of the gear will result in the same pair of teeth passing through mesh. If a fault exists on ***both*** the gear and pinion, the maximum effect on vibration will be noted when the respective faults of the pinion and gear enter mesh at the same time. These faults may be a result of the manufacturing process or possible mishandling.

This component is very low in frequency. Generally it is subaudible (<20 Hz), but as its nature is that of a beat frequency, it is often detected by ear. It is difficult to detect on a normal Fast Fourier Transform (FFT) spectrum, but fortunately it is readily apparent in the time domain signal obtained from a seismic type of transducer. Someone using a geardrive which exhibits the effects of tooth repeat frequency will generally complain of a low frequency repetitive "growl" from the drive. The frequency can be determined with a seismic transducer. In most cases, however, the frequency is low enough to be determined by audible count and a wristwatch with a seconds display.

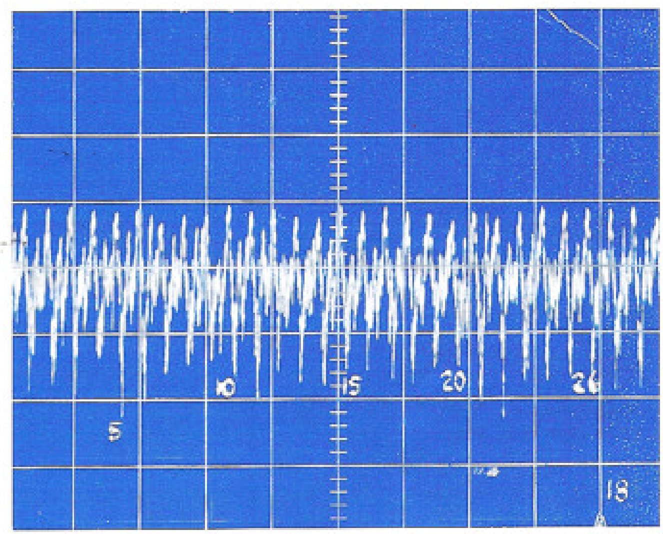

Figure 2
Time domain trace

The time domain trace (Figure 2) represents the output of a precision sound level meter. A gear drive operating at 5528 rpm input speed and 8334 rpm output speed had excessive pitch line runout in both a 98 tooth gear and a 65 tooth pinion. The problem was readily apparent as the audible noise of the gear drive fluctuated 6-8 dB with a mean sound pressure level of approximately 88 dBA. The frequency was determined to be 85 cpm using a wristwatch. The time domain trace was furnished as "hard copy" documentation of the problem. Mathematically, the tooth repeat frequency is:

$$f_{tr} = \frac{f_m \times N_a}{N_g \times N_p} \text{ (Hz)}$$

For this particular gear drive, from which the time domain was obtained, the tooth repeat frequency was calculated to be 1.42 Hz or 85 cpm. Using the oscilloscope trace (Figure 2), the frequency was calculated at approximately 88 cpm. (26.3/18 x 60 = 88 cpm). ►

Since (N_a) is equal to unity for a true hunting tooth combination, mathematically the tooth repeat frequency for a true hunting tooth combination is expressed as:

$$f_{tr} = \frac{f_{rg}}{N_p} \text{(Hz)}$$

It is important to realize that this expression does not apply to a gearset with multiple assembly phases.

Relationship between frequency components

The tabular form "Fundamental Gearset Frequencies" (Table 1) is based on the 15 by 9 tooth gearset. This form, or a similar form, should be part of the documentation involving any gear-related vibration problem.

Fundamental gearset frequencies (example)

Gearset Data:

1. Number of teeth on pinion	(N_p)	9
2. Rotating speed of pinion, rpm	(R_p)	3000
3. Number of teeth on gear	(N_g)	15
4. Rotating speed of gear, rpm	(R_g)	1800
5. Ratio, N_g/N_p or R_p/R_g	(M_g)	1.667

Determination of number of assembly phases (N_a):

6. Factors of number of gear teeth
   $F_1 \times F_2 \times F_3 \times F_4 = N_g$ 1, 3, 5, 15, ____
7. Factors of number of pinion teeth
   $F_1 \times F_2 \times F_3 \times F_4 = N_p$ 1, 3, 3, 9, ____
8. Product of prime factors ***common*** to gear and pinion
   N_a = 1 × 3 × ____ × ____ = 3

Calculation of gearset fundamental frequencies:

9. Pinion rotational frequency, f_{rp} (Hz) $f_{rp} = R_p/60$	50
10. Gear rotational frequency, f_{rg} (Hz) $f_{rg} = R_g/60$	30
11. Mesh frequency, f_m (Hz) $f_m = f_{rp} \times N_p = f_{rg} \times N_g$	450
12. Tooth repeat frequency, f_{tr} (Hz) $f_{tr} = (f_m \times N_a)/(N_g \times N_p)$	10
13. Assembly phase passage frequency, f_a (Hz) $f_a = f_m/N_a$	150

Does the relationship $f_{tr} \le f_{rg} \le f_{rp} \le f_a \le f_m$ hold true?

10 ≤ 30 ≤ 50 ≤ 150 ≤ 450

If not, recheck 6 through 13!

Table 1

The expression $f_{tr} \le f_{rg} \le f_{rp} \le f_a \le f_m$ appears at the bottom of Table 1. This statement is true for any two gears in mesh. In proving this statement, it becomes readily apparent that many of the frequencies are related. Table 2 summarizes the relationships between frequencies for any two gears in mesh.

Relationships between gear frequencies

To Obtain		f_{tr}	f_{rg}	f_{rp}	f_a	f_m
Multiply						
f_{rp}	BY	N_a/N_g	$1/M_g$	1	N_p/N_a	N_p
f_{rg}		N_a/N_p	1	M_g	N_g/N_a	N_g
f_m		$N_a/(N_g \times N_p)$	$1/N_g$	$1/N_p$	$1/N_a$	1

Where:

N_a = Number of assembly phases
N_p = Number of teeth on pinion
N_g = Number of teeth on gear
M_g = Ratio
f_{tr} = Tooth repeat frequency, (Hz)
f_{rg} = Gear rotational frequency, (Hz)
f_{rp} = Pinion rotational frequency, (Hz)
f_a = Assembly phase passage frequency, (Hz)
f_m = Mesh frequency, (Hz)

Table 2

Conclusion

The effects of the rotational components and their identification are generally understood by the vibration analyst. The actual gear mesh and its vibratory components may be difficult to establish. The dynamics of the mesh are nonlinear and are heavily influenced by many variables. Sum and difference frequencies are very likely to occur. Hopefully, this article will at least help to identify the basic frequencies occurring during operation of gear drives. ■

References:

1. Joseph E. Shigley, ***Mechanical Engineering Design***
2. M.F. Spotts, ***Design of Machine Elements.***
3. American Petroleum Institute, ***"Special Purpose Gear Drives For Refinery Service."***